

4 Variable Karnaugh Map

		W X			
		00	01	11	10
Y Z	00				
	01				
	11				
	10				

$W X Y Z = 0 1 1 0$ $W X Y' Z$

- The map of four variables is shown beside. It consists of sixteen small squares, arranged as a 4 x 4 small squares. Each small square represents one minterm. Each minterm has four neighboring (or adjacent) minterms.
- The map minimization of four variable functions depends on combining adjacent squares. The combination of adjacent squares help in the simplification of the functions.
 - One square represents minterm (4 literals).
 - Two adjacent squares represent a term with three literals.
 - Four adjacent squares represent a term of two literals.
 - Eight adjacent squares represent a term of one literal.
 - Sixteen adjacent squares represent the function ($F = 1$)

Boolean Algebra to Karnaugh Map

Plot:

$$A'B + BD + AB'C'D + B'CD$$

		A B			
		C D	00	01	11
00	0	0	1	0	0
01	0	0	1	1	1
11	1	1	1	1	1
10	0	0	1	0	0

Karnaugh Map to Boolean Algebra (Simplification)

Plot:

$$A'B + BD + AB'C'D + B'CD$$

Generate:

$$A'B + CD + AD$$

		A B			
		C D	00	01	11
00	0	0	1	0	0
01	0	0	1	1	1
11	1	1	1	1	1
10	0	0	1	0	0

Complement Functions

The Complement of a Karnaugh Map is formed by inverting every location.

		L M			
		00	01	11	10
N P	00	1	0	0	1
	01	1	1	1	1
	11	1	1	1	0
	10	1	1	1	1

$F \Rightarrow F'$

		L M			
		00	01	11	10
N P	00	0	1	1	0
	01	0	0	0	0
	11	0	0	0	1
	10	0	0	0	0

Finding a Function Through Product of Sums

Sometimes it is easier to plot the Product of Sums:

$$F' = MN'P' + LM'NP$$

$$F = (M' + N + P) (L' + M + N' + P')$$

		L M			
		N P	00	01	11
N P	00	1	0	0	1
	01	1	1	1	1
	11	1	1	1	0
	10	1	1	1	1

Karnaugh Map with “Don’t Care” Values

When we design combinational logic circuits, we sometimes encounter situations where combinations of the input variables will never occur. In such cases, we can assume that these conditions can take on the value 1 or 0, whichever going to give us a simpler expression. These conditions are indicated by letter X or D.

Simplify:

$F(A,B,C,D)$

$$= \sum m(1, 5, 6, 10, 11, 14) \\ + \sum d(0, 7, 9, 15)$$

$$= AC + BC + A'C'D$$

A B \ C D		A B			
		00	01	11	10
C D	00	X ₀			
	01	1 ₁	1 ₅		X ₉
	11		X ₇	X ₁₅	1 ₁₁
	10		1 ₆	1 ₁₄	1 ₁₀

Minimum Sum of Products

- A minimum sum of products is
 - a sum of products expression which
 - has a minimum number of terms
 - has a minimum number of literals for all expressions with a minimum number of terms

Minimum

		A B			
		00	01	11	10
C D	00				1
	01			1	1
	11		1	1	1
	10		1	1	1

$$F = AB' + BC + AD$$

		A B			
		00	01	11	10
C D	00				1
	01			1	1
	11		1	1	1
	10		1	1	1

$$F = AB' + BC + ABC'D$$

Minimum Product of Sums

- A minimum product of sums is
- a product of sums expression which
- has a minimum number of products
- has a minimum number of literals for all expressions with a minimum number of products

$$F = (A + B) (B + C) (A' + D) \quad \Leftarrow$$

$$F = (A + B) (B + C) (A' + D) (B + D)$$